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## Strong Efficient Domination in Jump Graphs

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**ABSTRACT:** Let  $G(V,E)$  be a simple graph. E.Sampath kumar and L.Puspalatha introduced the concept of strong and weak domination in graph [5]. In this paper this concept is extended to efficient domination in jump graphs. A subset  $S$  of  $V(J(G))$  is called strong (weak) efficient dominating set of  $J(G)$  if every  $v \in V(J(G))$ .  $|N_s(v) \cap S| = 1$  ( $|N_w(v) \cap S| = 1$ ) where  $N_s(v) = \{u \in V(J(G)) \mid uv \in E(J(G)), \deg(u) \geq \deg(v)\}$  and  $N_w(v) = N_s(v) \cup \{v\}$ ,  $N_w(v) = N_w(v) \cup \{v\}$ . The minimum cardinality of a strong(weak) efficient dominating set is called strong (weak) efficient dominating number of  $J(G)$  and is denoted by  $\gamma_{se}(J(G))$  ( $\gamma_{we}(J(G))$ ).

A graph is strong efficient if there is a strong efficient dominating set. In this paper we find classes of graphs which are strong efficient and compare strong efficient dominating number with  $\gamma_s(J(G))$ ,  $\Gamma_s(J(G))$ ,  $i_s(J(G))$  and  $\beta_s(J(G))$

**KEYWORDS:** strong efficient domination number, Full degree vertex, strong and weak neighbors.

### I. INTRODUCTION

Throughout this paper we consider finite, undirected simple graphs. Let  $G=(V,E)$  be a graph. The degree of any vertex  $u$  in  $G$  is the number of edges incident with  $u$  and is denoted by  $\deg u$ .

The minimum and maximum degree of a vertex is denoted by  $\delta(G)$  and  $\Delta(G)$  respectively. A vertex of degree  $g$  is called an isolated vertex and a vertex of degree 1 is called a pendent vertex, for all graph theoretic terminologies and notations we follow Harary [4]. The following definitions are necessary for the present study.

1.1 **Definition:** A subset  $M$  of  $E(J(G))$  is called a matching in  $J(G)$  if its elements are edges and no two are adjacent in  $J(G)$  the two ends of an edge in  $M$  are said to be matched under  $M$ . A matching  $M$  saturates a vertex  $v$  and  $v$  is said to be  $m$ -saturated. If every vertex of  $J(G)$  is  $M$ -saturated then the matching  $M$  is called perfect.

1.2 **Definition:** A subset  $S$  of  $V(J(G))$  of a jump graph  $J(G)$  is called a dominating set if every vertex in  $V(J(G)) - S$  is adjacent to a vertex in  $S$ . The domination number  $\gamma(J(G))$  is the minimum cardinality of a dominating set of  $J(G)$ .

1.3 **Definition :** A subset  $S$  of  $V(J(G))$  is called a strong dominating set of  $J(G)$  if every  $v \in V(J(G)) - S$ , there exists  $u \in S$  such that  $u$  and  $v$  are adjacent and  $\deg u \geq \deg v$ .

1.4 **Definition:** An subset  $S$  of  $V(J(G))$  is called as efficient dominating set of  $G$  if every  $v \in V(J(G))$   $|N(v) \cap S| = 1$

A set of points is independent if no two points are adjacent.

A strong dominating set of  $J(G)$  which is also independent is called independent strong dominating set of  $J(G)$ .

The minimum cardinalities of the independent strong dominating set of  $J(G)$  is called an independent strong domination number of  $J(G)$  and is denoted by  $i_s(J(G))$ . The maximum cardinality of an independent strong domination number of  $J(G)$  is denoted by  $\beta_s(J(G))$ . Motivated by these definitions, we have defined a new type of domination called strong efficient jump graph, if it admits a strong efficient dominating set. The minimum cardinality of a strong efficient dominating set is called the strong efficient domination number of  $J(G)$  and is denoted by  $\gamma_{se}(J(G))$ .

In this paper we classify graphs which are strong efficient and compare strong efficient domination number with  $\gamma_s(J(G))$ ,  $\Gamma_s(J(G))$ ,  $i_s(J(G))$  and

$\beta_s(J(G))$  we have proved that there are graphs in which several strong efficient dominating sets of different cardinalities exist which is interesting in this sense that all efficient dominating sets have the same cardinality  $\gamma_{se}$ .

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## II. MAIN RESULTS

**Definition 2.1:** Let  $J(G) = (V, E)$  be a jump graph. a sub set of  $V(J(G))$  is called a strong(weak)efficient dominating set of  $J(G)$  if for every  $v \in V(J(G))$   $|N_s[v] \cap S| = 1$  ( $|N_w[v] \cap S| = 1$ ) where  $N_s(v) = \{ u \in V(J(G)) ; uv \in E(J(G)), \deg u \geq \deg v \}$  and

where  $N_w(v) = \{ u \in V(J(G)) ; uv \in E(J(G)), \deg v \geq \deg u \}$

$N_s(v) = N_s(v) \cup \{v\}$  and  $N_w(v) = N_w(v) \cup \{v\}$ . The minimum cardinality of a strong (weak) efficient dominating set of  $G$  is called the strong (weak) efficient domination number of  $G$  is strong efficient if and only if there exists a strong efficient dominating set of  $J(G)$ . Not all graphs admit strong efficient dominating set.

**Theorem 2.1;**  $\sqrt_s(J(G)) \leq \sqrt_{se}(J(G))$

**Proof:** Let  $S$  be a minimum strong efficient dominating set of  $J(G)$ .

Let  $v \in J(V) - S$  then  $|N_s[v] \cap S| = 1$

i.e., there exists  $u \in S$  such that  $u$  and  $v$  are adjacent and  $\deg u \geq \deg v$  therefore  $S$  is a strong dominating set of  $J(G)$ , therefore  $\sqrt_s(J(G)) \leq \sqrt_{se}(J(G))$

**Theorem 2.2.** Every strong efficient dominating set is independent.

**Proof;** Let  $S$  be a strong efficient dominating set

i.e.,  $u, v \in S$  suppose  $u$  and  $v$  are adjacent let without loss of generality  $\deg u \geq \deg v$  then  $|N_s[v] \cap S| \geq 2$  a contradiction. Therefore  $S$  is independent.

**Theorem 2.3.** If  $S$  is a strong efficient dominating set of a connected graph  $J(G)$  then  $J(V) - S$  is a dominating set of  $J(G)$ .

**Proof:** Since every strong efficient dominating set is independent and  $G$  is connected, every vertex in  $S$  is adjacent to atleast one vertex in  $J(V)-S$ .

Therefore  $J(V)-S$  is a dominating set of  $J(G)$ .

**Remark 2.4.** If  $S$  is a strong efficient dominating set of  $J(G)$  then  $J(V)-S$  need not be a weak dominating set of  $J(G)$ .

**Observation 2.5.** Let  $J(G)$  be a connected jump graph with at least 2 pendent vertices and  $\sqrt_{se}(J(G)) = 1$ . Then  $J(G)$  has no perfect matching.

( since  $\sqrt_{se}(J(G)) = 1$  implies that  $J(G)$  has a full degree vertex)

**Theorem 2.6;** Let  $J(G)$  be a connected graph at  $|V(J(G))| = n$  ( $n$  is even) if

$\sqrt_{se}(J(G)) > \frac{n}{2}$  then  $J(G)$  has no perfect matching.

**Proof:** Let  $S$  be a  $\sqrt_{se}$ -set of a connected jump graph  $J(G)$  as  $|V(J(G))| = n$ .

Let  $|S| = t$  since  $\sqrt_{se}(J(G)) > \frac{n}{2}$ ,  $t > \frac{n}{2}$

Suppose that  $G$  has a perfect matching  $M$ . Then every vertex of  $J(G)$  is  $M$ -saturated. Therefore every vertex of  $S$  is  $M$ -saturated. Since  $S$  is independent every line of  $M$  has either one end in  $S$  and other end in  $V - S$  or both ends in  $V - S$ . Since  $M$  is a perfect matching  $|M| = \frac{n}{2}$  But  $|S| = t > \frac{n}{2}$ . Therefore

$t - \frac{n}{2}$  vertices of  $S$  and  $M$ -unsaturated, a contradiction. Therefore  $J(G)$  has no perfect matching.

Note:  $i_s(J(G)) \leq \sqrt_{se}(J(G)) \leq \beta_o(J(G))$

**Definition 2.7[4]** A regular spanning sub graph of degree 1 is called 1-factor (1F)

**Theorem 2.8.**  $K_{n,n}$ -1F is strong efficient and  $\sqrt_{se}(K_{n,n}-1F) = 2 \quad \forall n \in \mathbb{N}$

**Proof:** Let  $G = K_{n,n}-1F$

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Let  $V(J(G)) = \{v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$  Since we remove 1F from  $K_{n,n}$  degree of each vertex is reduced to  $n-1$ .

Each  $v_i$  is not adjacent to one  $u_j \forall i=1$  and  $\forall j=1$  to  $n$  Since  $\{u_i, u_j\}$  is a strong efficient dominating set.

Hence  $\sqrt{se}(K_{n,n}-1F) = 2 \forall n \in \mathbb{N}$

**Theorem 2.9.**  $[K_n]$  is a strong efficient and  $\sqrt{se}(J(K_n)) = \frac{n^2-3n+4}{2}$  where

$$p = |V(J(K_n))|$$

**Proof:** let  $n \geq 3$ , let  $v_1, v_2, v_3, \dots, v_n$  be the vertices of  $K_n$ . let  $J(G) = J(K_n)$ .

$V(J(K_n)) = \{v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_{(n/2)}\}$ . By definition of  $[K_n]$  each  $u_i$  is adjacent to exactly 2 vertices in  $K_n$ .

Therefore  $|V(J(K_n))| = p = n + nC_2 = n + \frac{n(n-1)}{2} = \frac{n^2-n}{2}$

$\Delta(J(G)) = \deg v_i$  for any  $i, i=1$  to  $n$ .

Each  $v_i$  is adjacent to the remaining  $(n-1)$   $v_i$ 's and  $(n-1)$   $u_i$ 's

$$\therefore \Delta(J(G)) = (n-1) + (n-1) = 2n-2$$

$$\text{Total number of } u_i \text{'s} = nC_2 = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$$

Therefore number of  $u_j$ 's which are not adjacent to  $v_i = \frac{n^2-n}{2} - (n-1)$

$$= \frac{n^2-3n+1}{2} \text{ these } u_j \text{'s together with } v_i \text{'s form a strong efficient dominating set } S \text{ of}$$

$J(G)$ .

$$\therefore \sqrt{se}(J(G)) \leq \frac{n^2-3n+1}{2}$$

Let  $T$  be any strong efficient dominating set of  $J(k_n)$ . Since  $T$  is independent,  $T$  can contain atmost one  $v_i$ 's  $1 \leq i \leq n$ . Since

for any  $n \geq 3$ , no  $u_j$  can strongly dominate any  $v_i$   $T$  contains atleast one  $v_i$  ( $1 \leq i \leq n$ ) Therefore  $T$  contains exactly one  $v_i$ .

Any  $u_j$  can dominate only two  $v_i$ 's and all  $u_j$ 's are independent Therefore  $T$  contain all  $u_j$ 's not adjacent with the  $v_i \in T$  therefore  $|T| \geq 1 + \frac{n^2-n}{2} - (n-1)$

$$= \frac{n^2-3n+4}{2}$$

$$\text{Therefore } \sqrt{se}(J(K_n)) \geq \frac{n^2-3n+4}{2}$$

$$\text{Hence } \sqrt{se}(J(K_n)) = \frac{n^2-3n+4}{2}$$

Where  $n=2$ ,  $[K_n] = C_3$ ,  $\sqrt{se}(J(C_3))=1$

$$\text{Hence } \sqrt{se}(J(K_2)) = \frac{2^2-3 \cdot 2+4}{2}$$

$$\text{Then Hence } \sqrt{se}(J(K_n)) = \frac{n^2-3n+4}{2}$$

$$\begin{aligned} \text{Also } p - \Delta(J(G)) &= \frac{n^2+n}{2} - (2n-2) = \frac{n^2-n-4n+4}{2} \\ &= \frac{n^2-3n+4}{2} \end{aligned}$$

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Therefore  $\sqrt[2]{\delta(J(G))} = p - \Delta(J(G))$

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